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STATIC LOAD-CARRYING CAPACITY OF CIRCULAR RIGID-PLASTIC PLATES UNDER GAUSSIAN DISTRIBUTION OF PRESSURE

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Abstract—Static load-carrying capacities of circular rigid-plastic plates subjected to Gaussian distributions of pressure are presented in this paper. The effects of varying the load distribution as well as the boundary conditions are determined. Curves are presented which bound the load-carrying capacity for boundary conditions between the ideal cases of clamped and simply supported.

INTRODUCTION

This paper presents the results of an analysis of the static load-carrying capacity of circular rigid-plastic plates subjected to Gaussian distribution of pressure. Curves are presented which bound the load-carrying capacity for boundary conditions between the ideal cases of clamped and simply supported. The static load-carrying capacity of the plate is defined as that load which produces initial plastic yielding throughout the plate with the resulting stress state satisfying the small-deflection bending equations of equilibrium.

Historically, there have been very few published analyses of plastic plates with loads other than point, uniform, or linear distribution. The only general distribution of load which has received significant analytical attention is the Gaussian distribution. By varying a single parameter, this general distribution can span the extremes of the point to uniform distribution. This versatility was recognized by Sneddon[1] who approximated the dynamic loading of a projectile on a thin, infinite elastic plate by a Gaussian distribution of pressure. Madden[2] related this loading to the initial velocity distribution on a second plate located a set distance behind a projectile-penetration first plate. The first study of this loading on a plastic plate was by Thomson[3]. He obtained the solution of a rigid, perfectly plastic plate of material obeying the Tresca yield condition subjected to an initial impulse of Gaussian distribution. Weidman[4], in considering the response of simply supported circular plastic plates to distributed time-varying loadings, presented an example case of a radial Gaussian distribution of pressure with an exponential decay. The plate material was also rigid, perfectly plastic obeying the Tresca yield condition. In an analysis by Hayduk[5], the plate was considered rigid, viscoplastic, obeying the von Mises yield condition, and was subjected to either a Gaussian distribution of pressure or impulse.

The foregoing analyses treated the dynamic response of plates to Gaussian loads. The dynamic solution of [5] also required prior knowledge of the static load-carrying capacity because of the linearization technique used in the dynamic analysis. This requirement provided the initial impetus to the work reported herein.

ANALYSIS

Shown in Fig. 1 is a circular plate of radius R and thickness $2h$ with either simply supported or clamped boundary conditions and subjected to a Gaussian distribution of pressure. The pressure distribution with magnitude p_0 at the plate center decreases radially at a rate dependent on the magnitude of the parameter β in the exponential term. In the limit as $\beta \to 0$ the distribution becomes uniform over the plate and as $\beta \to \infty$ the distribution becomes concentrated at the center. For a particular load distribution specified by a positive value of β, the load-carrying capacity of the plate is defined as the value p_0' for which the equilibrium equations and the yield function are satisfied simultaneously at all points of the plate. Since the load-carrying capacity rather than the stress distribution is of primary interest, the solution method of Eason[6] is used

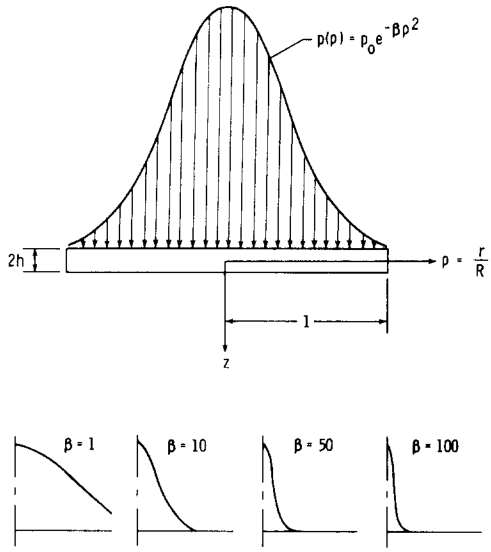

Fig. 1. Circular plate with Gaussian distribution of pressure and distributions for various values of the parameter β.

because the load-carrying capacity can be determined without consideration of the stress field. The analysis following Eason[6] is briefly presented in the following paragraphs.

The small deflection equations of equilibrium for a circular plate with a Gaussian distribution of pressure,

$$\frac{\mathrm{d}}{\mathrm{d}r}(rQ) + rp_0 \mathrm{e}^{-a^2r^2} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}r}(rM_r) - M_\phi = rQ \tag{1}$$

and the Hill[7] radial and circumferential bending moment relations,

$$M_r = -\frac{2}{\sqrt{(3)}} M_0 \sin\left(\frac{1}{6}\pi - \theta\right)$$

$$M_\phi = \frac{2}{\sqrt{(3)}} M_0 \sin\left(\frac{1}{6}\pi + \theta\right) \tag{2}$$

which satisfy the von Mises yield condition,

$$M_r^2 - M_r M_\phi + M_\phi^2 - M_0 = 0 \tag{3}$$

can be combined into the single equation

$$r \cos\left(\frac{1}{6}\pi - \theta\right)\frac{\mathrm{d}\theta}{\mathrm{d}r} - \cos\theta = \frac{\sqrt{(3)}p_0}{4M_0}\left(\frac{\mathrm{e}^{-a^2r^2} - 1}{a^2}\right). \tag{4}$$

M_0 is the fully plastic bending moment of the plate material and θ is a variable defining the stress distribution. At the center of the plate $M_r = M_\phi = M_0$ and θ has the value $\pi/2$. At the edge of the plate θ has the value $\pi/6$ for simple-support boundary conditions ($M_r = 0$) or $-(\pi/3)$ for clamped boundary conditions.

Defining nondimensional parameters $s = r^2/R^2$, $F = \sqrt{(3)}(p_0R^2/4M_0)$, and $\beta = a^2R^2$, eqn (4) can be recast into the form

$$2s \cos\left(\frac{1}{6}\pi - \theta\right)\frac{\mathrm{d}\theta}{\mathrm{d}s} - \cos\theta = -\frac{F}{\beta}(1 - \mathrm{e}^{-\beta s}). \tag{5}$$

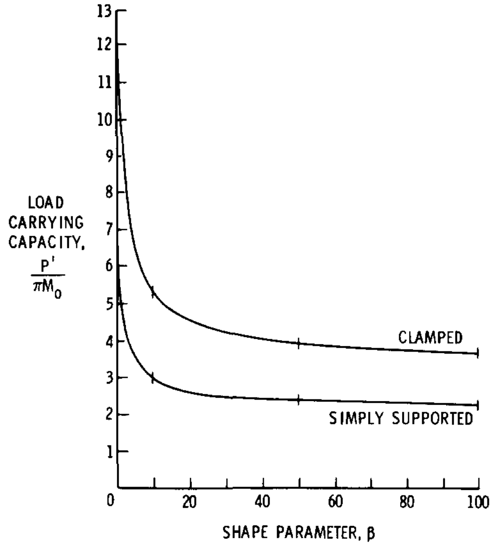

Fig. 2. The effect of load distribution on the load-carrying capacity of simply supported and clamped circular rigid plastic plates.

For a particular value of β the nondimensional central pressure magnitude, F' (value of F corresponding to the collapse load), is found by numerically integrating eqn (5) over the range $\pi/2 \geq \theta \geq \pi/6$ for simple-support boundary conditions or $\pi/2 \geq \theta \geq -\pi/3$ for clamped boundary conditions.

RESULTS AND CONCLUSIONS

The results of this numerical integration for both boundary conditions are presented graphically in Fig. 2 where the ordinate parameter $P'/\pi M_0$ is the total nondimensional load-carrying capacity of a circular plate obtained by integrating the pressure distribution over the plate

$$\frac{P'}{\pi M_0} = \frac{4}{\sqrt{(3)}} F' \frac{1 - e^{-\beta}}{\beta}. \tag{6}$$

The sensitivity of the load-carrying capacity to changes in load distribution over the range $0 \leq \beta \leq 10$ is clearly exhibited by both the simple-support and clamped boundary condition cases. For $\beta = 10$ the load-carrying capacity is less than half that for the uniform distribution in both cases. From the sketches in Fig. 1, one can see that for $\beta = 10$ the load is almost entirely within the central region of radius $R/2$. As the load becomes more and more concentrated at the center from β of 50 to infinite, the load-carrying capacity $P'/\pi M_0$ gradually approaches the known point load values of 2 for the simple-support case and $4/\sqrt{(3)}$ for the clamped case determined by Hopkins and Wang[8]. At the other end of the load spectrum, the results for the uniform loading ($\beta = 0$) were in agreement with the known results of [8] – 6.51 for the simple-support case and 12.55 for the clamped case. For a general Gaussian distribution of pressure on a circular plate, the curves in Fig. 2 bound the load-carrying capacity for all uniform and continuous boundary conditions between the ideal cases of clamped and simply supported.

The static collapse load results presented herein should not be construed as lower bounds of limit analysis theory because the velocity fields were not obtained. The solution is believed to be exact because Eason's methods[6] for the uniform load case can be used to obtain corresponding velocity fields for these Gaussian distribution cases. However, these velocity fields were not needed for the corresponding dynamic problem[5]. The linearization technique used for the dynamic problem required knowledge only of the static collapse load which was used to eliminate unknown static collapse moment distributions appearing in the governing equations.

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